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COMPUTATION OF THE ORBIT OF PLANET

(558)

BY

J. H. SCARBOROUGH

Submitted for the Degree of Doctor of Philosophy
(Ph.D.) to Vanderbilt University,
Nashville, Tenn.

May, 1908

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The New Era Printing Company
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COMPUTATION OF THE ORBIT OF PLANET (558).

The small planet (558) was discovered by Wolf at Königstuhl-Heidelberg, February 9, 1905, and was reported in the *Astronomische Nachrichten*, Volume 167, page 208, as follows:

"*Photographische Aufnahmen von kleinen Planeten.*

1905. Q,B, M. Z. Kgst. 11^h 51.9^m, Feb. 9, 1905.

α 9^h 47.5^m

δ $+13°$ $4'.0$

Gr. 12.0

Tag. Bewegungen, -0.9^m $+4'.0$.

Q,B, und**** sind neue Planeten.

M. Wolf."

The value of Tag. Bewegungen as given in the above report is corrected on page 303 of the same volume, and should read, for declination, $+7'$ instead of $+4'$. As reported by the same observer, page 349 of the same volume, another observation on March 13, 1905, gives:

M. Z. Kgst. 11^h 0.5^m

α 9^h 25.5^m

δ $16°$ $24'$

The following observations made by Palisa at Wien (k. k. Sternwarte) are reported in Volume 168 of the *Astr. Nach.*:

1905	M. Z. Wien	α app.	l. par. Δ	δ app.	l. par. Δ
	h m s	h m s		° ′ ″	
Mar. 29	9 47 31	9 21 16.01	8.970	+17 20 12.9	0.656
" 31	9 37 20	9 21 9.24	8.955	+17 24 37.7	0.655
Apr. 4	10 17 35	9 21 13.23	9.265	+17 31 50.4	0.667
" 9	9 33 14	9 21 50.70	9.154	+17 37 34.3	0.659
" 23	10 37 14	9 26 38.49	9.491	+17 36 23.6	0.705
" 30	8 50 38	9 30 29.76	9.286	+17 27 2.5	0.670
May 10	9 27 18	9 37 34.88	9.465	+17 4 15.8	0.702

The places for the two Heidelberg observations given on the preceding page are also given at another place in the *Astr. Nach.* (M. Z. Kgst.) :

	h m	h m s	° ′ ″
Feb. 9	11 51.5	9 47 51.19	+13 4 23.50
Mar. 13	11 0.5	9 25 35.30	+16 25 34.90

I have examined the complete files of the *Astr. Nach.* from the date of the discovery of this planet up to December 8, 1905, a period of more than six months after the last observation given in Vol. 168. So the above appears to be a complete list of all the observations made during the opposition period immediately following the discovery.

This planet was numbered (558), September 26, 1905, by F. Bauschinger of the Astron. Recheninstitut, Berlin ; and this designation, (558), will be used in any future reference to it in this paper.

The Provisional Elements of (558) have been computed by A. Berberich of the Astron. Recheninstitut, Berlin, and are published in Vol. 169, p. 285, of the *Astr. Nach.*, as follows

Epoch, Feb. 9.5 (M. T. Berlin) 1905.

M		41°	17′	34″.4
ω		314	40	6 .0
Ω		144	15	43 .8
i		8	21	3 .0
ϕ		2	14	1 .0
μ	715″.481			
log a	0 .463606.			

These elements are referred to the mean equinox of 1905.0.

It is my purpose in this thesis : To construct, from the provisional elements just given, a provisional ephemeris for the opposition period from February 9 to May 10, 1905. To correct the provisional elements by comparing the observed places, already given, with the computed places to be shown in the provisional ephemeris.

I. Computation of Provisional Ephemeris.

(*a*) *Eccentric Anomaly.* — To determine the eccentric anomaly, E, I used the transcendental equation

$$M = E - e \sin E. \tag{1}$$

Assuming $E = M$, a first approximation was found; using the first approximation and repeating the process, a second approximation was determined; the second approximation was corrected by means of the formula

$$\Delta E' = \frac{M - E' + e \cdot \sin E'}{1 - e \cdot \cos E'} \tag{2}$$

and the results were found to satisfy the given equation (1).

The results are given below:

Date, Gr. M. T.	1″ Approx.	2″ Approx.	$\Delta E'$	E
Feb. 9.5	42° 46′ 30.″81	42° 49′ 4.″62	4.″53	42° 49′ 9.″15
" 13.5	43 35 36.09	43 38 10.30	4.50	43 38 14.80
" 17.5	44 24 40.31	44 27 14.84	4.42	44 27 19.26
" 21.5	45 13 43.49	45 16 18.18	4.39	45 16 22.57
" 25.5	46 2 45.59	46 5 20.35	4.32	46 5 24.67
Mar. 1.5	46 51 46.61	46 54 21.29	4.24	46 54 25.53
" 5.5	47 40 46.56	47 43 21.02	4.17	47 43 25.19
" 9.5	48 29 45.36	48 32 19.52	4.09	48 32 23.61
" 13.5	49 18 43.04	49 21 16.75	4.02	49 21 20.77
" 17.5	50 7 39.56	50 10 12.73	3.92	50 10 16.65
" 21.5	50 56 34.95	50 59 7.40	3.86	50 59 11.26
" 25.5	51 45 29.13	51 48 0.82	3.75	51 48 4.57
" 29.5	52 34 22.15	52 36 52.90	3.68	52 36 56.58
Apr. 2.5	53 23 13.95	53 25 43.69	3.56	53 25 47.25
" 6.5	54 12 4.51	54 14 33.13	3.47	54 14 36.60
" 10.5	55 00 53.93	55 3 21.23	3.37	55 3 24.60
" 14.5	55 49 42.07	55 52 7.96	3.27	55 52 11.23
" 18.5	56 38 28.93	56 40 53.33	3.18	56 40 56.51
" 22.5	57 27 14.54	57 29 37.35	3.06	57 29 40.41
" 26.5	58 15 58.86	58 18 19.94	2.94	58 18 22.88
" 30.5	59 4 41.91	59 7 1.13	2.84	59 7 3.97
May 4.5	59 53 23.66	59 55 40.93	2.73	59 55 43.66
" 8.5	60 42 4.06	60 44 19.29	2.63	60 44 21.92
" 12.5	61 30 43.18	61 32 56.23	2.54	61 32 58.77

All the values of E given in the above table have been verified by substituting in the given equation (1).

(*b*) *True Anomaly, Radius Vector, and Longitude.* — The true anomaly, V, the radius vector, r, and the longitude of the planet in its orbit, u, were determined from the following:

$$\sqrt{r} \cdot \sin \frac{V}{2} = \sqrt{2a} \cdot \sin (45° + \tfrac{1}{2}\phi) \cdot \sin \frac{E}{2},$$

$$\sqrt{r} \cdot \cos \frac{V}{2} = \sqrt{2a} \cdot \cos (45° + \tfrac{1}{2}\phi) \cdot \cos \frac{E}{2},$$

$$u = V + (\pi - \Omega),$$

$$\pi = \omega + \Omega = 458° \; 55' \; 49''.8.$$

The results are shown in the following table:

Date, Gr. M. T.		V			u			log r
		°	′	″	°	′	″	
Feb.	9.5	44	21	34.00	359	1	40.00	0.451 010
"	13.5	45	12	3.66	359	52	9.66	0.451 180
"	17.5	46	2	30.68	0	42	36.68	0.451 350
"	21.5	46	52	55.18	1	33	1.18	0.451 530
"	25.5	47	43	17.54	2	23	23.54	0.451 706
Mar.	1.5	48	33	37.14	3	13	43.14	0.451 886
"	5.5	49	23	54.54	4	4	0.54	0.452 066
"	9.5	50	14	9.10	4	54	15.10	0.452 250
"	13.5	51	4	20.74	5	44	26.74	0.452 438
"	17.5	51	54	30.56	6	34	36.56	0.452 626
"	21.5	52	44	36.98	7	24	42.98	0.452 818
"	25.5	53	34	41.14	8	14	47.14	0.453 010
"	29.5	54	24	42.74	9	4	48.74	0.453 204
Apr.	2.5	55	14	41.56	9	54	47.56	0.453 400
"	6.5	56	4	37.64	10	44	43.64	0.453 600
"	10.5	56	54	31.14	11	34	37.14	0.453 798
"	14.5	57	44	21.20	12	24	27.20	0.454 004
"	18.5	58	34	8.02	13	14	14.92	0.454 206
"	22.5	59	23	53.88	14	3	59.88	0.454 414
"	26.5	60	13	36.08	14	53	42.08	0.454 618
"	30.5	61	3	15.00	15	43	21.00	0.454 830
May	4.5	61	52	51.26	16	32	57.26	0.455 042
"	8.5	62	42	25.06	17	22	31.06	0.455 250
"	12.5	63	31	54.90	18	12	0.90	0.455 468

(*c*) *The Constants of Gauss.* — The constants of Gauss were determined by means of the formulas:

$$\tan N = \frac{\tan i}{\cos \Omega}.$$

$$\cot A = \tan \Omega \cos i, \qquad \sin a = \frac{\cos \Omega}{\sin A},$$

$$\cot B = \frac{\cos i \cdot \cos (N + e)}{\tan \Omega \cos N \cdot \cos e}, \qquad \sin b = \frac{\sin \Omega \cos e}{\sin B},$$

$$\cot C = \frac{\cos i \cdot \sin (N + e)}{\tan \Omega \cos N \cdot \sin e}, \qquad \sin c = \frac{\sin \Omega \sin e}{\sin C},$$

Verification formula $\tan i = \dfrac{\sin b \cdot \sin c \cdot \sin (C - B)}{\sin a \cdot \cos A}$.

The results follow:

$$A = 234^\circ\ 33'\ 4''.32,$$

$$B = 146\ \ 00\ 18\ .22,$$

$$C = 128\ \ 43\ 19\ .32,$$

$$\log \sin a = 9.998\ 433,$$

$$\log \sin b = 9.981\ 523,$$

$$\log \sin c = 9.474\ 126.$$

These results were verified by the verification formula. They were used in the determination of the heliocentric coördinates of the planet, as shown in the next section.

(*d*) *Heliocentric Coördinates.* — The heliocentric coördinates x, y, z, were found from the following:

$$x = r \cdot \sin a \cdot \sin (A + u),$$

$$y = r \cdot \sin b \cdot \sin (B + u),$$

$$z = r \cdot \sin c \cdot \sin (C + u),$$

r being the radius vector of the planet, u the longitude of the planet in its orbit, and a, b, c, A, B, C, the constants of Gauss already determined.

The results are given in the table on page 6 :

(*e*) *Rectangular Coördinates of the Sun.* — The rectangular

coördinates of the sun, X, Y, Z (referred to the mean equinox of 1905.0), are taken from the Nautical Almanac. I do not consider it necessary to reproduce them here.

Date		x	y	z
Feb.	9.5	−2.264 979	+1.551 557	+0.665 492
"	13.5	−2.290 179	+1.519 403	+0.658 111
"	17.5	−2.314 879	+1.486 907	+0.650 587
"	21.5	−2.339 139	+1.454 153	+0.642 938
"	25.5	−2.362 872	+1.421 055	+0.635 147
Mar.	1.5	−2.386 117	+1.387 669	+0.627 221
"	5.5	−2.408 856	+1.353 984	+0.619 164
"	9.5	−2.431 106	+1.320 026	+0.610 981
"	13.5	−2.452 856	+1.285 816	+0.602 674
"	17.5	−2.474 088	+1.251 297	+0.594 237
"	21.5	−2.494 806	+1.216 544	+0.585 681
"	25.5	−2.515 006	+1.181 528	+0.577 000
"	29.5	−2.534 678	+1.146 271	+0.568 201
Apr.	2.5	−2.553 835	+1.110 772	+0.559 280
"	6.5	−2.572 467	+1.075 051	+0.550 249
"	10.5	−2.590 550	+1.039 100	+0.541 098
"	14.5	−2.608 135	+1.002 951	+0.531 846
"	18.5	−2.625 156	+0.966 580	+0.522 477
"	22.5	−2.641 663	+0.930 022	+0.513 003
"	26.5	−2.657 577	+0.893 256	+0.503 420
"	30.5	−2.672 994	+0.856 328	+0.493 741
May	4.5	−2.687 850	+0.819 213	+0.483 959
"	8.5	−2.702 149	0.781 925	+0.474 070
"	12.5	−2.715 919	+0.744 500	+0.464 096

(*f*) *Geocentric Coördinates of the Planet.*—The geocentric coördinates of the planet, right ascension, declination, and the distance from the center of the earth were determined by the formulas:

$$x + X = \rho \cos \delta \cos \alpha,$$

$$y + Y = \rho \cos \alpha \sin \delta,$$

$$z + Z = \rho \sin \delta.$$

These coördinates were computed directly for every four days from February 9, 1905, to May 12, 1905, and the intermediate values found by interpolation.

Thus I have obtained the desired provisional ephemeris which is given in the table on next page.

PROVISIONAL EPHEMERIS OF PLANET (558), FEBRUARY 9 TO MAY 12, 1905.

Date		α			δ			log ρ
		h	m	s	°	′	″	
Feb.	9.5	9	47	51.21	13	4	34.63	0.264 742
"	10.5	9	47	3.43	13	11	40.53	
"	11.5	9	46	15.55	13	18	46.17	
"	12.5	9			13	25	51.41	
"	13.5	9	44	39.29	13	32	56.09	0.264 448
"	14.5				13	40	0.68	
"	15.5	9	43	3.26	13	47	3.21	
"	16.5							
"	17.5	9	41	27.73	14	1	3.78	0.265 261
"	18.5							
"	19.5	9	39	53.10	14	14	54.46	
"	21.5	9	38	19.85	14	28	32.67	0.267 181
"	23.5	9	36	48.47	14	41	56.42	
"	25.5	9	35	19.36	14	55	2.35	0.270 147
"	27.5	9	33	52.91	15	7	47.53	
Mar.	1.5	9	32	29.56	15	20	10.48	0.274 126
"	3.5	9	31	9.80	15	32	9.12	
"	5.5	9	29	53.88	15	43	41.36	0.279 043
"	7.5	9	28	42.06	15	54	44.65	
"	9.5	9	27	34.73	16	5	18.23	0.284 836
"	10.5	9	27	2.88	16	10	23.42	
"	11.5	9	26	32.30	16	15	20.75	
"	12.5	9	26	2.96	16	20	10.13	
"	13.5	9	25	34.92	16	24	51.41	0.291 401
"	14.5	9	25	8.21	16	29	24.41	
"	15.5	9	24	42.84	16	33	49.11	
"	16.5	9	24	18.82	16	38	5.49	
"	17.5	9	23	56.17	16	42	13.42	0.298 638
"	19.5	9	23	15.03	16	50	3.62	
"	21.5	9	22	39.52	16	57	19.61	0.306 456
"	23.5	9	22	9.77	17	4	1.03	
"	25.5	9	21	45.80	17	10	8.01	0.314 761
"	26.5	9	21	36.00	17	12	58.59	
"	27.5	9	21	27.66	17	15	40.55	
"	28.5	9	21	20.78	17	18	13.92	
"	29.5	9	21	15.38	17	20	38.65	0.323 468
"	30.5	9	21	11.47	17	22	54.70	
"	31.5	9	21	9.02	17	25	2.15	
Apr.	1.5	9	21	8.04	17	27	1.08	
"	2.5	9	21	8.54	17	28	51.49	0.332 496
"	3.5	9	21	10.51	17	30	33.38	
"	4.5	9	21	13.95	17	32	6.85	
"	5.5	9	21	18.83	17	33	31.96	
"	6.5	9	21	25.16	17	34	48.77	0.341 774
"	7.5	9	21	32.93	17	35	57.33	
"	8.5	9	21	42.12	17	36	57.71	
"	9.5	9	21	52.72	17	37	49.95	
"	10.5	9	22	4.74	17	38	34.11	0.351 210
"	11.5	9	22	18.17	17	39	10.21	
"	12.5	9	22	32.99	17	39	38.34	
"	13.5				17	39	58.61	

PROVISIONAL EPHEMERIS OF PLANET (558), FEBRUARY 9 TO MAY 12, 1905.—*Continued.*

Date		α	δ	log ρ
		h m s	° ′ ″	
Apr.	14.5	9 23 6.70	17 40 11.10	0.369 758
"	15.5		17 40 15.90	
"	16.5	9 23 45.73	17 40 13.06	
"	17.5	9	17 40 2.67	
"	18.5	9 24 29.98	17 39 44.72	0.370 341
"	19.5		17 39 19.30	
"	20.5	9 25 19.33	17 38 46.53	
"	21.5	9 25 45.87	17 38 6.44	
"	22.5	9 26 13.61	17 37 19.32	0.379 923
"	23.5	9 26 42.53	17 36 25.20	
"	24.5	9 27 12.63	17 35 24.09	
"	25.5	9 27 43.91	17 34 15.95	
"	26.5	9 28 16.34	17 33 0.82	0.389 453
"	27.5	9 28 49.95	17 31 38.62	
"	28.5	9 29 24.69	17 30 9.52	
"	29.5	9 30 0.54	17 28 33.65	
"	30.5	9 30 37.49	17 26 51.10	0.398 912
May	1.5	9 31 15.52	17 25 2.03	
"	2.5	9 31 54.61	17 23 6.42	
"	3.5	9 32 34.75	17 21 4.29	
"	4.5	9 33 15.92	17 18 55.68	0.408 252
"	5.5	9 33 58.13	17 16 40.58	
"	6.5	9 34 41.34	17 14 19.12	
"	7.5	9 35 25.53	17 11 51.42	
"	8.5	9 36 10.68	17 9 17.57	0.417 450
"	9.5	9 36 56.77	17 6 37.63	
"	10.5	9 37 43.79	17 3 51.69	
"	11.5	9 38 31.71	17 0 59.83	
"	12.5	9 39 20.52	16 58 2.13	0.426 481

In the computation of the ephemeris just given, the method of differences was used to detect errors in the results. The errors thus found were corrected by a recomputation.

A portion of the ephemeris is given for every two days, instead of for every day, as the values for the omitted dates were not needed in determining the computed coördinates to be compared with the observed coördinates.

II. To Find Correction for the Provisional Elements.

(*a*) *Comparison of Computed with Observed Places.* — For purposes of comparison the times of observation at Heidelberg

and at Vienna were freed from aberration and reduced to Greenwich mean time. In making this reduction I used the constants for the K. K. Universitats-Sternwarte in Wien, as given in the *Annalen der K. K. Sternwarte zu Wien*, Vol. XV, p. 113. I give them below:

Geocentric latitude	48°	2′	55″.4
Longitude (from Greenwich)	— 1^h	5^m	21^s.49, or
	— 16°	20′	22″.3

Log of dist. from center of earth 9.999 211

The corresponding constants for Heidelberg are:

Geocentric latitude	49°	23′	2″.55
Longitude (from Gr.)	— 0^h	34^m	48^s.5

Log dist. from center of earth 9.999 075

Using the ephemeris given in Part I of this thesis, I found by interpolation the coördinates of the planet for the Greenwich mean time of observation corrected for aberration. The results are given in the table below. The coördinates given in the table are the true coördinates of the place of the planet referred to the center of the earth.

COMPUTED α AND δ AT TIMES OF OBSERVATION.

Date (Gr. M. T. Freed from Abbr.)		α			δ		
	°	h	m	s	°	′	″
Feb.	9.4598	9	47	53.13	13	4	17.52
Mar.	13.4211	9	25	37.08	16	24	29.52
"	29.3505	9	21	16.09	17	20	17.66
"	31.3432	9	21	9.31	17	24	42.74
Apr.	4.3710	9	21	13.62	17	31	55.26
"	9.3398	9	21	50.93	17	37	42.12
"	23.3832	9	26	39.09	17	36	31.88
"	30.3086	9	30	30.23	17	27	11.11
May	10.3334	9	37	35.89	17	4	19.75

For convenenience in comparison with the computed coördinates, the observed coördinates were corrected for parallax in right ascension and declination, and reduced to the mean equinox of 1905.0.

The parallax factors, given on page 2, must each be divided by the distance from the planet to the earth at a given date, in order to obtain the correction for parallax. The parallax factors were not given for the two Heidelberg observations, but I computed them by the usual formulas.

The following table gives the corrections to be applied; and the arrangement is such that:

"I" is correction for parallax in right ascension.

"II" is correction in right ascension for mean equinox of 1905.0.

"III" is correction for parallax in declination.

"IV" is correction in declination for mean equinox of 1905.0.

Date		I	II	III	IV
		s		"	"
Feb.	9.4598	−0.035	+0.067	+2.83	+4.94
Mar.	13.4211	+0.047	−0.128	+4.79	+6.32
"	29.3505	+0.044	−0.228	+2.15	+6.86
"	31.3432	+0.043	−0.240	+2.13	+6.88
Apr.	4.3710	+0.085	−0.239	+2.14	+6.85
"	9.3398	+0.064	−0.261	+2.05	+7.08
"	23.3832	+0.128	−0.356	+2.10	+7.56
"	30.3086	+0.077	−0.405	+1.87	+7.67
May	10.3334	+0.111	−0.484	+1.98	+8.09

These corrections were applied to the apparent coördinates given on page 2. The results are given in the table below. The values of α and δ in this table are the "observed" values after the corrections are applied. In the column marked "$0 - C, \alpha$" the difference between the "observed" and "computed" values of α are given, the sign being in the order of observed minus computed. In the column marked "$0 - C, \delta$" the differences between the observed and computed values of declination are given.

Computed values are given on page ~~11~~ 9

It will be noticed that the two Heidelberg observations give much larger differences than the others. This would seem to indicate that there is some error not accounted for. The re-

Date		α			δ			O—C, α Δα	O—C, δ Δδ
		h	m	s	°	′	″	s	″
Feb.	9.4598	9	47	51.22	13	4	31.27	−1.91	+13.65
Mar.	13.4211	9	25	35.22	16	24	46.01	−1.86	+16.49
"	29.3505	9	21	15.83	17	20	21.91	−0.26	+ 4.25
"	31.3432	9	21	9.04	17	24	46.71	−0.27	+ 3.97
Apr.	4.3710	9	21	13.08	17	31	59.39	−0.54	+ 4.13
"	9.3398	9	21	50.51	17	37	43.43	−0.42	+ 1.31
"	23.3832	9	26	38.26	17	36	33.26	−0.83	+ 1.38
"	30.3086	9	30	29.43	17	27	12.04	−0.80	+ 0.93
May	10.3334	9	37	34.52	17	4	25.87	−1.37	+ 0.12

ports of these two observations given in the *Astr. Nach.* give the time only to the tenths of a minute. This is not sufficient to account for the large differences referred to; yet, as a part of the report gives only approximate values, it is possible that the coördinates given are only approximate values rather than carefully measured results. Not having time to communicate with the observer at Heidelberg, I decided to leave out these two observations and base the remainder of my computations on the other observations.

(*b*) *Normal Places.* — Omitting the two Heidelberg observations, I collected the others into three groups:

Group I, observations of March 29 and 31.

Group II, observations of April 4, 9 and 23.

Group III, observations of April 30 and May 10.

Taking the average of the differences for each group as the correction for the computed place at the beginning of the day nearest the mean date, we have the following results:

Group	Date	Average		Normal Places	
		Δα	Δδ	α	δ
				h m s	° ′ ″
I	Mar. 30.5	−0.27	+4.11	9 21 11.20	17 22 58.81
II	Apr. 14.5	−0.60	+2.27	9 23 6.10	17 40 13.37
III	May 5.5	−1.08	+3.82	9 33 57.05	17 16 44.40

(*c*) *Computation of Corrections of the Elements.* — To compute the corrections for the elements, I first referred the position of the planet to the ecliptic, and computed the latitude and

longitude at the times given in the table for normal places. After taking account of the latitude of the sun, and computing and checking in the usual manner, I found the coördinates to be :

Date	Latitude	Longitude
Mar. 30.5	1° 48′ 10″.348	137° 16′ 20″.83
Apr. 14.5	2 13 00.160	137 37 7.46
May 5.5	2 39 31.170	140 12 0.05

It will be noticed that the latitudes of the planet are small, and the angle between two of the radii vectores still smaller. Slight errors in observations, or in certain approximations would have considerable effect. Moreover, after continuing the work at some length, I found that in certain portions of the work it was necessary to carry the computation beyond seven decimal places ; and I did not have a complete table of more than seven places. So, after spending considerable time and labor, I found that it was necessary to change my plan, and refer the planet to the equator as the plane of reference.

I then used the differential method, and found the changes that would be produced in the elements by the variations in right ascension and declination given on page 20. By differentiating the equations involving the elements and the coördinates considered as variables, formulas were obtained giving the relations between the variations in the elements and the variations in the coördinates. These differentiations, after some reduction to more convenient forms, give,

1° For variation in right ascension,

$$\cos\delta\frac{d\alpha}{d\pi}\Delta\pi + \cos\delta\frac{d\alpha}{d\Omega}\Delta\Omega + \cos\delta\frac{d\alpha}{di}\Delta i + \cos\delta\frac{d\alpha}{d\phi}\Delta\phi + \cos\delta\frac{d\alpha}{dM}\Delta M + \cos\delta\frac{d\alpha}{d\mu}\Delta\mu = \cos\delta\cdot\Delta\alpha.$$

2° For variation in declination,

$$\frac{d\delta}{d\pi}\Delta\pi + \frac{d\delta}{d\Omega}\Delta\Omega + \frac{d\delta}{di}\Delta i + \frac{d\delta}{d\phi}\Delta\phi + \frac{d\delta}{dM}\Delta M + \frac{d\delta}{d\mu}\Delta\mu = \Delta\delta.$$

In order to obtain the six equations between the variations of the six elements, I took the combined results of the variations of the coördinates given for the normal places ; the three changes in right ascension giving me three equations, and the three changes in declination giving the other three.

I might have taken each observation separately, and from the seven observations obtained fourteen equations, and then combined these into six equations. By either method of obtaining the six equations, all the observations available are made use of.

The equations of condition for finding the values of the variations of the elements were thus found to be :

+1.275012	$\Delta\pi$—0.045347	$\Delta\Omega$+0.035849	Δi+1.895022	$\Delta\phi$+1.347753	ΔM— 1.752088	$\Delta\mu$= 3″.8650
—0.196062	—0.187308	+0.213273	—0.250601	—0.209490	+13.394215	= 4 .110
+1.161339	—0.041779	+0.044644	+1.778783	+1.225739	+ 5.436958	= 8 .5754
—0.175604	—0.171191	+0.262144	—0.229251	—0.187798	+13.018983	= 2 .270
+1.025810	—0.038474	+0.057220	+1.654673	+1.078990	+20.764357	=15 .469
—0.164980	—0.149625	+0.313806	—0.234041	—0.175896	+ 9.383532	= 3 .820

The second member of each of the odd-numbered equations above is $\cos\delta\cdot\Delta\alpha$, while the second member of each of the others is $\Delta\delta$. The second member in each is expressed in seconds of arc.

To verify the computations of the coefficients in the above equations, I assumed a small arbitrary change in each element, namely :

$$\text{Let } \Delta\pi = -20''$$
$$\Delta\Omega = -10$$
$$\Delta i = +10$$
$$\Delta\phi = +10$$
$$\Delta M = +10$$
$$\Delta\mu = +00.01$$

Applying these changes to the provisional elements, and using the elements thus corrected, I computed the right ascension of the planet for April 14.5 and compared it with

the right ascension already computed from the elements before the changes were applied. I found the difference to be,

$$\Delta\alpha = 8.48''.$$

I also found the value of $\Delta\alpha$ directly from the third equation in the list, using the same date, and the agreement was close enough to indicate that the computation of the coefficients in this equation is correct. As the computation of the places in this verification is rather long, I did not apply it to all the equations. However, it is very probable that if an error should occur in the computation of the coefficients of any one of the equations, it would affect the coefficients in the others also.

The solution of the six equations given above, involving the variations of the six elements, gives

$$\begin{aligned}
\Delta M &= +\ 3'\ 19''.49 \\
\Delta\phi &= +\ 1'\ 46''.15 \\
\Delta i &= +\ 4'.80 \\
\Delta\Omega &= +\ 8''.33 \\
\Delta\pi &= -\ 6'\ 5''.99 \\
\Delta\mu &= -\ 0''.001386.
\end{aligned}$$

I found $\Delta\alpha$ from the formula, $\Delta = \alpha - \frac{2}{3}\frac{\Delta\mu}{\mu}$,

the result being, $\Delta\alpha = 0.00000376.$

Applying the corrections just found to the provisional elements, we have the followiug:

$$\begin{aligned}
M &= 41^\circ\ 20'\ 53''.89 \\
\pi &= 98\ 49\ 43\ .81 \\
\Omega &= 144\ 15\ 52\ .13 \\
i &= 8\ 21\ 8\ .80 \\
\phi &= 2\ 15\ 47\ .15 \\
\mu &= 715\ .4796
\end{aligned}$$

Note that π, used above, is equal to $\Omega + \omega$ (used on page 3).

The epoch is the same as on page 3, and the elements are referred to the mean equinox of 1905.0.

These are the corrected elements sought.

The approximate time of the second opposition is May 17, 1906, the time of first opposition being February 4, 1905.

From the mean daily motion as corrected, the sidereal period is found to be 1811.37 days; the synodic period can be readily found by comparing with the period of the earth—it is nearly 457 days.

The perturbations of Jupiter and Saturn have not been taken into account. This topic would of itself form an interesting one for a thesis.

Conclusion.— I cannot hope that the corrected elements here given are free from error. The best results of computations based upon the observations of one opposition only, cannot in any case give us results altogether free from error. The short period over which the available observations extend, is such as to prevent results altogether accurate. Some other difficulties have already been pointed out.

I selected this planet from a large list of newly discovered asteroids after examining all the available observations of each one. This afforded me the best material at hand for the work I desired to accomplish, for the observations were more numerous and extended over a longer period than those of any other one in the list.

The fact that all computation and checking of results had to be done by one person has made the task much longer than I had anticipated. However, I hope to be able at some future time to carry the work forward to other oppositions, as they are observed, and thus make still further corrections of the elements. We cannot hope for results of extreme accuracy, without carrying the work forward through several oppositions.

In the prosecution of the work, I cannot claim originality as to the theory and methods used. In fact, the theory of the

computation of orbits of bodies moving about the sun as given by Gauss in his "Theoria Motus" a century ago, is the basis of all the work of the computation of orbits by the astronomers of the present day.

I have consulted freely, and used formulas, from the following:

"Theoria Motus," by Gauss, translation by Davis.

"Lecons sur la Determination des Orbits," par Tisserand, arranged by Perchot.

"Les Orbits des Cometes et Planetes," by D'Oppolzer, French translation.

"Tafeln zur Theoretischen Astronomie," by Bauschinger.

"Theoretical Astronomy," by Watson.

Astronomische Nachrichten, complete files to December, 1905.

I am under special obligation to Wm. J. Vaughn, LL.D., head of the Department of Mathematics and Astronomy, Vanderbilt University, for his many valuable suggestions, and also for the free use of his excellent private library.

I am also under special obligation to Professor D. T. Wilson, of the Case School of Applied Science (Cleveland, O.), who has kindly verified most of the computations in this thesis.